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Bosonization of the two coupled spinon–holon chains

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Abstract. The two coupled spinon–holon chains are investigated. One finds that for the set of parameters which allows us to map the model into two coupled Hubbard chains tunnelling occurs and the Luttinger liquid is destroyed. In addition we find that $t_{\perp}^{2}/J_{\perp} \gg t_{\|}^{2}/J_{\|}$ preserves the Luttinger liquid.

1. Introduction

The physics of strongly correlated electrons is described by the Hubbard model in the $t/U \to 0$ limit. This limit is equivalent to the t–J model. In one dimension the t–J model is equivalent to a spinon–holon model [1] which is best described within the $U(1)$ gauge formulation [2].

For the Hubbard model it was suggested [1] that, due to the orthogonality catastrophe, single-particle tunnelling is prohibited between two Luttinger chains. This result has been contested in [3–5].

In the present paper we investigate the two coupled holon–spinon chains. We find that for the generic case which corresponds to two coupled Hubbard chains single-particle and two-particle tunnelling is allowed in agreement with [3]. In the language of the renormalization group the transverse hopping parameter $t_{\perp}$ is a relevant variable.

For the particular case of parameters $(t_{\perp}^{2}/J_{\perp}) \gg (t_{\|}^{2}/J_{\|})$ one finds that Anderson's idea of confinement is realized. As a result the Luttinger liquid is not destroyed.

This situation is realized when the velocity of the antisymmetric charge density modes vanishes, $U_{c}^{(-)} = 0$.

Formally this problem is investigated with the aid of the new bosonization method introduced in [2]. The tunnelling is investigated with the aid of the Coulomb gas formalism [6–8].

2. The $U(1)$ spinon–holon model

We consider the two-chain t–J model.

$$H = H_{\|} + H_{\perp} \tag{1a}$$

$$H_{\|} = \sum_{r=1,2} \left\{ -t_{\|} \sum_{x} \sum_{\sigma=\uparrow\downarrow} C_{\sigma}^{+}(r,x) C_{\sigma}(r,x+d) + \text{HC} \right.$$
$$\left. + J_{\|} \sum_{x} [S(r,x) \cdot S(r,x+d) - N(r,x)N(r;x+d)] \right\} \tag{1b}$$

$$H_\perp = -t_\perp \sum_x \sum_{\sigma=\uparrow,\downarrow} C_\sigma^+(1,x)C_\sigma(2,x) + \mathrm{HC} + J_\perp \sum_x [S(1,x)\cdot S(2,x) - N(1,x)N(2,x)]. \tag{1c}$$

$H_\parallel$ represents the intrachain $t_\parallel$–$J_\parallel$ model and $H_\perp$ represents the interchain $t_\perp$–$J_\perp$ model ($r = 1.2$ is the chain index and d is the lattice constant). For the remaining part we use the slave boson representation given in [2]: $C_\sigma^+(r,x) = b(r,x)f^+(r,x)$, $C_\sigma(r,x) = b^+(r,x)f_\sigma(r,x)$, $b^+(r,x)b(r,x) + \sum_{\sigma=\uparrow,\downarrow} f_\sigma^+(r,x)f_\sigma(r,x) = 1$, where b, b^+ are holon bosons and f_σ, f_σ^+ are spinon fermions.

Following the method given in [1], we replace the Hamiltonian given in (1) by a $U(1)$ gauge spinon–holon model:

$$H = H_\parallel + H_\perp \tag{2a}$$

$$H_\parallel = \int \mathrm{d}x \sum_{r=1,2} \left\{ \frac{1}{2m_\mathrm{b}} |(\partial_x + \mathrm{i}a_\parallel(r,x))b^+(r,x)|^2 + \sum_{\sigma=\downarrow,\uparrow} \frac{1}{2m_f} |(\partial_x + \mathrm{i}a_\parallel)f_\sigma|^2 + a_0(r,x)[b^+(r,x)b(r,x) + \sum_{\sigma=\uparrow,\downarrow} f_\sigma^+(r,x)f_\sigma(r,x) - 1] + \tfrac{1}{2}\hat{V}_\parallel |b^+(r,x)b(r,x+d)|^2 \right\}. \tag{2b}$$

$$H_\perp = \int \mathrm{d}x \left\{ -\frac{\tilde{J}_\perp}{2} \sum_{\sigma=\uparrow,\downarrow} f_\sigma^+(1,x)\mathrm{e}^{\mathrm{i}a_\perp(x)} f_\sigma(2,x) + \mathrm{HC} - \tilde{t}_\perp b^+(1,x)\mathrm{e}^{\mathrm{i}a_\perp(x)} b(2,x) + \mathrm{HC} + \tfrac{1}{2}\hat{V}_\perp |b^+(1,x)b(2,x)|^2 \right\}. \tag{2c}$$

For the remaining part we will study the model given in equations (2a)–(2c). This is a $U(1)$ gauge model with the gauge fields a_0, $a = (a_\perp, a_\parallel)$. The model given in equations (2b) and (2c) depends on the original parameters, $t_\parallel$, $J_\parallel$, $t_\perp$, $J_\perp$ in the following way:

$$\hat{V}_\parallel = \frac{4t_\parallel^2}{J_\parallel} - J_\parallel \qquad \hat{V}_\perp = \frac{4t_\perp^2}{J_\perp} - J_\perp. \tag{2d}$$

$$\frac{1}{2m_\mathrm{b}} = \mathrm{d}^2 t_\parallel \left\langle \left| \sum_{\sigma=\uparrow,\downarrow} f^+(r,x)f_\sigma(r,x+d) + \frac{2t_\parallel}{J_\parallel} b^+(r,x)b(r;x+d) \right| \right\rangle. \tag{2e}$$

$$\tilde{t}_\perp = \mathrm{d}^2 t_\perp \left\langle \left| \sum_{\sigma=\uparrow,\downarrow} f_\sigma^+(1,x)f_\sigma(1,x) + \frac{2t_\perp}{J_\perp} b^+(1,x)b(2,x) \right| \right\rangle. \tag{2f}$$

$$\frac{1}{m_\mathrm{f}} = \frac{1}{m_\mathrm{b}} \left(\frac{J_\parallel}{t_\parallel} \right) \qquad \frac{\tilde{J}_\perp}{2} = \tilde{t}_\perp \left(\frac{J_\perp}{2t_\perp} \right). \tag{2g}$$

The zero-component gauge field a_0 enforces the charge conservation and $a_\parallel$, $a_\perp$ the current conservation of spinons and holons.

3. Bosonization of the model

In this section we will use the Jordan–Wigner method to bosonize the model given in equations (2*a*)–(2*c*). We will follow the method introduced in [2].

$$C(r,x) = b^{+}(r,x)f_{\sigma}(r,x) \qquad C^{+}(r,x) = f_{\sigma}^{+}(r,x)b(r,x) \tag{3a}$$

$$f_{\sigma}(r,x) = e^{i\chi(r,x)}\tilde{f}_{\sigma}(r,x) \tag{3b}$$

$$\tilde{f}_{\sigma}(r,x) = [e^{i\theta_{\sigma}(r,x)} + e^{-i\theta_{\sigma}(r,x)}]\eta_{\sigma}(r,x) \tag{3c}$$

$$b(r,x) = \eta_{0}(r,x)[1 + e^{i2\theta_{0}(r,x)} + e^{i2\theta_{0}(r,x)}]. \tag{3d}$$

$\eta_{\sigma}(r,x)$ and $\eta_{0}(r,x)$ are boson operators. $\theta_{\sigma}(r,x)$ is the one-dimensional Jordan–Wigner phase which maps the boson $\eta_{\sigma}(r,x)$ into fermion $\tilde{f}_{\sigma}(r,x)$. In order to have anticommutation relations between spinons in different chains we must add the two-dimensional Jordan–Wigner phase $\chi(r,x)$ in the discrete form. $2\theta_{0}(r,x)$ is the one-dimensional Jordan–Wigner phase which maps bosons into bosons. This phase becomes important only when we reach the metal–insulator transition and will be neglected here.

The one-dimensional Jordan–Wigner phase $\theta_{\sigma}(r,x)$ obeys the condition

$$\partial_{x}\theta_{\sigma}(r,x) = \pi f_{\sigma}^{+}(r,x)f_{\sigma}(r,x) = \pi\bar{\rho}_{\sigma}(r) + \pi J_{\tau,\sigma}(r,x) \tag{3e}$$

whenever the averaged density $\bar{\rho}_{\sigma}(r)$ and the density operator are defined by $\bar{\rho}_{\sigma}(r) = \langle f_{\sigma}^{+}(r,x)f_{\sigma}(r,x)\rangle \equiv K_{F}/\pi$, $J_{\tau,\sigma}(r,x) = f_{\sigma}^{+}(r,x)f_{\sigma}(r,x) - \langle f_{\sigma}^{+}(r,x)f_{\sigma}(r,x)\rangle$. The expectation value is taken with respect to the free fermion spinons with mass m_{f} and density $1-\delta$.

The Jordan–Wigner phase $\chi(r,x)$ obeys

$$\begin{aligned}\chi(1,x) &= \int_{x'\neq x} dx' \, \mathrm{Im}\ln[x - x' + id] \sum_{\sigma=\uparrow,\downarrow} f_{\sigma}^{+}(2,x')f_{\sigma}(2,x') \\ \chi(2,x) &= \int_{x'\neq x} dx' \, \mathrm{Im}\ln[x - x' - id] \sum_{\sigma=\uparrow,\downarrow} f_{\sigma}^{+}(1,x')f_{\sigma}(1,x').\end{aligned} \tag{3f}$$

d is the distance between the chains. In the limit $d \to 0$ we obtain $\partial_{x}\chi(1,x) = -\pi\sum_{\sigma=\uparrow,\downarrow} f_{\sigma}^{+}(2,x)f_{\sigma}(1,x)$ and $\partial_{x}\chi(2,x) = \pi f_{\sigma}^{+}(1,x)f_{\sigma}(1,x)$.

In the next step we construct the Euclidean action for the Hamiltonian given in equation (2). We introduce a many-body coherent state basis $|\eta\rangle = |\eta_{0},\eta_{\uparrow},\eta_{\downarrow}\rangle$. Using this coherent state basis we obtain the Euclidean action:

$$S = \int d\tau \, \{\langle\eta|\partial_{\tau}|\eta\rangle + \langle\eta|H|\eta\rangle\}. \tag{4a}$$

The action given in (4*a*) has the following explicit *bosonic* form:

$$S = \int dx \int d\tau \left\{ \sum_{r=1,2} [\eta_{0}^{*}(r;x,\tau)(\partial_{\tau} + ia_{0}(r;x,\tau))\eta_{0}(r;x,\tau) - ia_{0}(r;x,\tau) \right.$$
$$+ \sum_{\sigma=\uparrow,\downarrow}\sum_{\pm} \eta_{\sigma}^{*}(r;x,\tau)(\partial_{\tau} + ia_{0}(r;x,\tau) \pm i\partial_{\tau}\theta_{\sigma}(r;x,\tau)$$

$$+ \mathrm{i}\partial_\tau \chi(r;x,\tau))\eta_\sigma(r;x,\tau) + \frac{1}{2m_b}|(\partial_x + \mathrm{i}a_\parallel(r;x,\tau))\eta_0(r;x,\tau)|^2$$

$$+ \sum_{\sigma=\uparrow,\downarrow}\sum_{\pm}\frac{1}{2m_f}|(\partial_x + \mathrm{i}a_\parallel(r;x,\tau) \pm \mathrm{i}\partial_x\theta_\sigma(r;x,\tau)$$

$$+ \mathrm{i}\partial_x\chi(r;x,\tau))\eta_\sigma(r;x,\tau)|^2 + \tfrac{1}{2}\hat{V}_\parallel|\eta_0^*(r;x,\tau)\eta_0(r;x+d,\tau)|^2]$$

$$- \frac{\tilde{J}_\perp}{2}\sum_{\sigma=\uparrow,\downarrow}\eta_\sigma^*(1;x,\tau)\exp[\mathrm{i}(\theta_\sigma(1;x,\tau) - \theta_\sigma(2;x,\tau) + \chi(1;x,\tau)$$

$$- \chi(2;x,\tau) + a_\perp(x,\tau))]\eta_\sigma(2;x,\tau) + \mathrm{HC} - \bar{t}_\perp\eta_0^*(1;x,\tau)$$

$$\times \exp[\mathrm{i}a_\perp(x,\tau)]\eta_0(2;x,\tau) + \mathrm{HC} + \tfrac{1}{2}\hat{V}_\perp|\eta_0^*(1;x,\tau)\eta_0(2;x,\tau)|^2$$

$$- E_F(\eta_0^*(1;x,\tau)\eta_0(1;x,\tau) + \eta_0^*(2;x,\tau)\eta_0(2;x,\tau))\Big\}. \qquad (4b)$$

E_F is the chemical potential and the sum in (4b) over the chiral component $\pm\theta_\sigma(r;x,\tau)$ vanishes.

(4b) can be further simplified if we use the complex form of the bosonic coherent fields:

$$\eta_\alpha(r;x,\tau) = \sqrt{\rho_\alpha(r;x,\tau)}\exp[\mathrm{i}\varphi_\alpha(r;x,\tau)] \qquad \alpha = 0,\uparrow,\downarrow \qquad (4c)$$

where ρ_α is the density and φ_α is a single-valued phase. Using the representation given in equation (4c) we replace in the action (4b) the fields η_α, η_α^* by the charge density $J_{\tau,\alpha}(r;x,\tau) = \eta_\alpha^*(r;x,\tau)\eta_\alpha(r;x,\tau) - \bar{\rho}_\alpha$ and current $J_{x,\alpha}(r;x,\tau) = \eta_\alpha^*(r;x,\tau)\partial_x\eta_\alpha(r;x,\tau)$.

Using the representation given in equation (4c) we obtain for the action in equation (4b) the representation

$$S = S_\parallel + S_\perp \qquad (5a)$$

$$S_\parallel = \int \mathrm{d}x \int \mathrm{d}\tau \Bigg\{ \sum_{r=1,2}\Bigg[\sum_{\alpha=0,\uparrow,\downarrow} \mathrm{i}J_{\tau,\alpha}(r;x,\tau)(\partial_\tau\varphi_\alpha(r;x,\tau) + a_0(r;x,\tau)$$

$$+ \partial_\tau\chi(r;x,\tau)(1-\delta_{\alpha,0})) + \mathrm{i}J_{x,\alpha}(r;x,\tau)(\partial_x\varphi_\alpha(r;x,\tau) + \partial_x\chi(r;x,\tau)(1-\delta_{\alpha,0})$$

$$+ a_\parallel(r;x,\tau)) + \frac{m_\alpha}{2\bar{\rho}_\alpha}(J_{x,\alpha}(r;x,\tau))^2 + \frac{\pi^2}{2m_\alpha}\bar{\rho}_\alpha(J_{\tau,\alpha}(r;x,\tau))^2(1-\delta_{\alpha,0})\Bigg]$$

$$+ [\tfrac{1}{2}\hat{V}_\parallel(J_{\tau,0}(1;x,\tau))^2 + \tfrac{1}{2}\hat{V}_\parallel(J_{\tau,0}(2;x,\tau))^2 + \tfrac{1}{2}\hat{V}_\perp J_{\tau,0}(1;x,\tau)J_{\tau,0}(2;x,\tau)]\Bigg\}. \qquad (5b)$$

In equation (5b) we have replaced the kinetic part by the Hubbard–Stratonovici field $J_{x,\alpha}(r;x,\tau)$ coupled to a phase derivative. In addition we linearize the cubic term $(\pi^2/6m_f)(\rho_\sigma(r;x,\tau))^3$ obtained from the one-dimensional Jordan–Wigner phase $\theta_\sigma(r;x,\tau)$. The interchain part which gives rise to tunnelling is given by

$$S_\perp = \int \mathrm{d}x \int \mathrm{d}\tau \Bigg\{ \sum_{\sigma=\uparrow,\downarrow} -2\hat{J}_\perp\cos(\theta_\sigma(1;x,\tau) - \theta_\sigma(2;x,\tau))\cos(\varphi_\sigma(1;x,\tau)$$

$$- \varphi_\sigma(2;x,\tau) + \chi(1;x,\tau) - \chi(2;x,\tau) + a_\perp(x,\tau))$$

$$- 2\hat{t}_\perp\cos(\varphi_0(1;x,\tau) - \varphi_0(2;x,\tau) + a_\perp(x,\tau))\Bigg\} \qquad (5c)$$

where we have replaced $\tilde{J}_\perp$ and $\tilde{t}_\perp$ by $\hat{J}_\perp = \tilde{J}_\perp\bar{\rho}_0/2$, $\hat{t}_\perp = \tilde{t}_\perp\bar{\rho}_0$ with the property $\bar{\rho}_a = \bar{\rho}_\uparrow + \bar{\rho}_\downarrow = 1 - \delta$, $\bar{\rho}_0 \equiv \delta$. ($\delta$ is the hole concentration.) The investigation of the tunnelling part $S_\perp$ is performed by expanding the partition function in powers of $\hat{t}_\perp$ and $\hat{J}_\perp$.

$$\begin{aligned} Z = \int & DJ_{\tau,\alpha}(r;x,\tau)D\varphi_\alpha(r;x,\tau) \\ & \times \exp[-S_\parallel]\sum_{N=0}^{\infty}\frac{(2\hat{t}_\perp)^N}{N!}\sum_{M=0}\frac{(2\hat{J}_\perp)^M}{M!}\int \mathrm{d}^2X_1\ldots\mathrm{d}^2X_N\int \mathrm{d}^2Y_1\ldots\mathrm{d}^2Y_M \\ & \times\prod_{i=1}^{N}\{\cos(\varphi_0(1;\boldsymbol{X}_i)-\varphi_0(2;\boldsymbol{X}_i)+a_\perp(\boldsymbol{X}_i)\} \\ & \times\prod_{J=1}^{M}\left\{\sum_{\sigma=\uparrow,\downarrow}\cos(\theta_\sigma(1,\boldsymbol{Y}_J)-\theta_\sigma(2,\boldsymbol{Y}_J))\cos(\varphi_\sigma(1;\boldsymbol{Y}_J)-\varphi_\sigma(2;\boldsymbol{Y}_J)\right. \\ & \left. +\chi(1;\boldsymbol{Y}_J)-\chi(2;\boldsymbol{Y}_J)+a_\perp(\boldsymbol{Y}_J))\right\}. \end{aligned} \tag{5d}$$

Following Coulomb gas methods we introduce the electric charge density $Q_\alpha(x,\tau)$ and magnetic monopole density $S_\alpha(x,\tau)$

$$\begin{aligned} Q_\alpha(x,\tau) &= \sum_{i=1}^{2N} q_i^{(\alpha)}\delta(\boldsymbol{X}-\boldsymbol{X}_i) \qquad \boldsymbol{X}=(x,\tau) \\ S_\alpha(x,\tau) &= \sum_{i=1}^{2M} s_i^{(\alpha)}\delta(\boldsymbol{X}-\boldsymbol{X}_i) \end{aligned} \tag{5e}$$

where $q_i^{(\alpha)} = \pm 1$, $s_i^{(\alpha)} = \pm 1$, $\alpha = 0, \uparrow, \downarrow$.

Using the instanton changes [7] given in equation (5*e*) we replace equation (5*d*) by the action $\tilde{S}_\perp$.

$$\begin{aligned} \tilde{S}_\perp = \int \mathrm{d}x\int \mathrm{d}\tau & \left\{\sum_{\alpha=0,\uparrow,\downarrow}\mathrm{i}Q_\alpha(x,\tau)[\varphi_\alpha(1;x,\tau)-\varphi_\alpha(2;x,\tau)+a_\perp(x,\tau)\right. \\ & +(\chi(1;x,\tau)-\chi(2;x,\tau))(1-\delta_{\alpha,0})] \\ & \left. +\mathrm{i}S_\alpha(x,\tau)(1-\delta_{\alpha,0})[\theta_\alpha(1;x,\tau)-\theta_\alpha(2;x,\tau)]\right\}. \end{aligned} \tag{5f}$$

The charge densities $Q_\alpha(x,\tau)$ and $S_\alpha(x,\tau)$ are controlled by the 'fugacities' $t_\perp$ and $\hat{J}_\perp$.

For the remaining part we have to consider the action $S = S_\parallel + \tilde{S}_\perp$, where $\tilde{S}_\perp$ is given in equation (5*f*) and $S_\parallel$ in equation (5*b*).

4. The dual construction

We integrate out in equations (5*b*) and (5*f*) the phases $\varphi_\alpha(r;x,\tau)$ and the gauge fields a_0, $a_\parallel$, $a_\perp$.

The phase integration in the path integral gives the continuity equations with the tunnelling charges $Q_\alpha(r;x,\tau)$.

$$\partial_\tau J_{\tau,\alpha}(r;x,\tau)+\partial_x J_{x,\alpha}(r;x,\tau)=Q_\alpha(x,\tau)[\delta_{r,1}-\delta_{r,2}]. \tag{6a}$$

Integration of the a_0 component gives the constraint condition

$$J_{\tau,0}(r;x,\tau)+J_{\tau,\uparrow}(r;x,\tau)+J_{\tau,\downarrow}(r;x,\tau)=0. \tag{6b}$$

Integration of $a_\parallel$ gives

$$J_{x,0}(r;x,\tau)+J_{x,\uparrow}(r;x,\tau)+J_{x,\downarrow}(r;x,\tau)=0. \tag{6c}$$

Integration of the $a_\perp$ gauge field gives the condition for the tunnelling charges:

$$Q_0(x,\tau)+Q_\uparrow(x,t)+Q_\downarrow(x,\tau)=0. \tag{6d}$$

Using equations (6*b*)–(6*d*) we introduce

$$-Q_0=Q_c=Q_\uparrow+Q_\downarrow \qquad Q_s=Q_\uparrow-Q_\downarrow \tag{6e}$$

and

$$\begin{aligned}-J_{\tau,0}(r;x,\tau)&=J_{\tau,c}(r;x,\tau)=J_{\tau,\uparrow}(r;x,\tau)+J_{\tau,\downarrow}(r;x,\tau)\\ J_{\tau,s}(r;x,\tau)&=J_{\tau,\uparrow}(r;x;\tau)-J_{\tau,\uparrow}(r;x,\tau).\end{aligned} \tag{6f}$$

'c' corresponds to charge density and 's' to spin density.

We introduce a symmetric and antisymmetric mode

$$\begin{aligned}J^{(\pm)}_{\tau,\alpha}&=\frac{1}{\sqrt{2}}(J_{\tau,\alpha}(1;x,\tau)\pm J_{\tau,\alpha}(2;x,\tau))\\ J^{(\pm)}_{x,\alpha}&=\frac{1}{\sqrt{2}}(J_{x,\alpha}(1;x,\tau)\pm J_{x,\alpha}(2;x,\tau)).\end{aligned} \tag{6g}$$

The continuity equation (6*a*) is replaced by the equations

$$\partial_\tau J^{(+)}_{\tau,c(s)}(x,\tau)+\partial_x J^{(+)}_{x,c(s)}(x,\tau)=0. \tag{7a}$$

$$\partial_\tau J^{(-)}_{\tau,c(s)}(x,\tau)+\partial_x J^{(-)}_{x,c(s)}(x,\tau)=\sqrt{2}Q_{c(s)}(x,\tau). \tag{7b}$$

The solution for equations (7*a*) and (7*b*) can be obtained in terms of new bosonic fields: $\Phi^{(+)}_c$, $\Phi^{(-)}_c$, $\Phi^{(+)}_s$, $\Phi^{(-)}_s$.

$$\begin{aligned}J^{(+)}_{\tau,c}&=\epsilon\partial_x\Phi^{(+)}_c \qquad & J^{(+)}_{x,c}&=-\epsilon\partial_\tau\Phi^{(+)}_c\\ J^{(+)}_{\tau,s}&=\epsilon\partial_x\Phi^{(+)}_s \qquad & J^{(+)}_{x,s}&=-\epsilon\partial_\tau\Phi^{(+)}_s\end{aligned} \tag{7c}$$

and

$$\begin{aligned}J^{(-)}_{\tau,c}&=\epsilon\partial_x\Phi^{(-)}_c+\sqrt{2}\frac{\partial_\tau Q_c}{\partial^2}\\ J^{(-)}_{x,c}&=-\epsilon\partial_\tau\Phi^{(-)}_c+\sqrt{2}\frac{\partial_x Q_c}{\partial^2}\\ J^{(-)}_{\tau,s}&=\epsilon\partial_x\Phi^{(-)}_s+\sqrt{2}\frac{\partial_\tau Q_s}{\partial^2}\\ J^{(-)}_{x,s}&=-\epsilon\partial_\tau\Phi^{(-)}_s+\sqrt{2}\frac{\partial_x Q_s}{\partial^2}.\end{aligned} \tag{7d}$$

ϵ is an arbitrary constant. We find that on choosing $\epsilon = \sqrt{2/\pi}$ the bosonic action takes a symmetric form.

The action S is entirely determined by the charges $J_{\tau,\alpha}(r; x, \tau)$ and currents $J_{x,\alpha}(r; x, \tau)$:

$$S = \int \mathrm{d}x \int \mathrm{d}\tau \left\{ \sum_{r=1,2} \left[\mathrm{i}\chi(r; x, \tau)(\partial_\tau J_{\tau,\mathrm{c}}(r; x, \tau) + \partial_x J_{x,\mathrm{c}}(r; x, \tau)) \right.\right.$$
$$+ \mathrm{i}Q_\mathrm{c}(x, \tau)\chi(r; x, \tau)(\delta_{r,1} - \delta_{r,2}) + \frac{m_\mathrm{f}}{2\bar{\rho}_\mathrm{c}}(J_{x,\mathrm{c}}(r; x, \tau))^2 + \frac{m_\mathrm{b}}{2\bar{\rho}_0}(J_{x,0}(r; x, \tau))^2$$
$$\left. + \frac{\pi^2}{2m_\mathrm{f}}\left(\frac{\bar{\rho}_\mathrm{c}}{4}\right)(J_{\tau,\mathrm{c}}(r; x, \tau))^2 + \frac{m_\mathrm{f}}{2\bar{\rho}_\mathrm{c}}(J_{x,\mathrm{s}}(r; x, \tau))^2 + \frac{\pi^2}{2m_\mathrm{f}}\left(\frac{\bar{\rho}_\mathrm{c}}{4}\right)(J_{\tau,\mathrm{s}}(r; x, t))^2 \right]$$
$$+ \tfrac{1}{2}\hat{V}_\parallel(J_{\tau,0}(1; x, \tau))^2 + \tfrac{1}{2}\hat{V}_\parallel(J_{\tau,0}(2; x, \tau))^2 + \tfrac{1}{2}\hat{V}_\perp J_{\tau,0}(1; x, \tau)J_{\tau,0}(2; x, \tau)$$
$$\left. + \mathrm{i}\pi S_\sigma(x, \tau)(\partial_x^{-1}J_{\tau,\sigma}(1; x, \tau) - \partial_x^{-1}J_{\tau,\sigma}(2; x, \tau)) \right\}. \tag{8}$$

Next we substitute the solutions given in equation (7*c*) and (7*d*) into equation (8). We find

$$S = S^{(+)} + S^{(-)} + S^{(\mathrm{Coulomb})} \tag{9a}$$

$$S^{(\pm)} = \int \mathrm{d}x \int \mathrm{d}\tau \left\{ \sum_{\beta=\mathrm{c,s}} \frac{U_\beta^{(\pm)}}{2g_\beta^{(\pm)}} \left[(\partial_x \Phi_\beta^{(\pm)}(x, \tau))^2 + \frac{1}{(U_\beta^{\pm})^2}(\partial_\tau \Phi_\beta^{(\pm)}(x, \tau))^2 \right] \right\} \tag{9b}$$

$$S^{(\mathrm{Coulomb})} = \int \mathrm{d}x \int \mathrm{d}x' \int \mathrm{d}\tau \int \mathrm{d}\tau' \left\{ \left[\frac{1}{2g_\mathrm{c}^{(-)}} Q_\mathrm{c}(x, \tau)Q_\mathrm{c}(x', \tau') \right.\right.$$
$$\left. + \frac{1}{2g_\mathrm{s}^{(-)}} Q_\mathrm{s}(x, \tau)Q_\mathrm{s}(x', \tau') + \frac{g_\mathrm{c}^{(-)}}{8} S_\mathrm{s}(x, \tau)S_\mathrm{s}(x', \tau') + \frac{g_\mathrm{s}^{(-)}}{8} S_\mathrm{s}(x, \tau)S_\mathrm{s}(x', \tau') \right]$$
$$\left. \times \hat{U}(x, \tau; x', \tau') + \mathrm{i}[S_\mathrm{c}(x, \tau)S_\mathrm{c}(x', \tau') + S_\mathrm{s}(x, \tau)S_\mathrm{s}(x', \tau')]\hat{\theta}(x, \tau; x', \tau') \right\} \tag{9c}$$

where $U_{\mathrm{c(s)}}^{(+)}$, $U_{\mathrm{c(s)}}^{(-)}$ are the velocities of the symmetric and antisymmetric mode

$$U_\mathrm{c}^{(\pm)} = \sqrt{\frac{(\pi^2/2m_\mathrm{f})(1-\delta) + (\hat{V}_\parallel \pm \hat{V}_\perp)}{m_\mathrm{f}/(1-\delta) + m_\mathrm{b}/\delta}} \qquad U_\mathrm{s}^{(\pm)} = \frac{K_\mathrm{F}}{m_\mathrm{f}} \tag{10a}$$

and the coupling constants for the charges $g_\mathrm{c}^{(\pm)}$ and spin $g_\mathrm{s}^{(\pm)}$ are given by

$$g_\mathrm{c}^{(\pm)} = \frac{2}{\sqrt{1 + F_\mathrm{c}^{(\pm)}}} \leqslant 2 \qquad g_\mathrm{s}^{(\pm)} = 2 \tag{10b}$$

with $F_\mathrm{c}^{(\pm)}$ given by

$$F_\mathrm{c}^{(\pm)} = 4\left[\frac{1-\delta}{\delta}\left(\frac{m_\mathrm{b}}{m_\mathrm{f}}\right) + (\hat{V}_\parallel \pm \hat{V}_\perp)\frac{1}{\pi^2}\left(\frac{m_\mathrm{f}}{1-\delta} + \frac{m_\mathrm{b}}{\delta}\right) \right] \tag{10c}$$

and $\hat{U}$ is the Coulombic potential:

$$\hat{U}(x, \tau; x', \tau') = \ln\left| \frac{\sqrt{(x-x')^2 + (\tau-\tau')^2}}{R} \right| \tag{10d}$$

$$\hat{\theta}(x, \tau; x', \tau') = -\tan^{-1}\left(\frac{\tau-\tau'}{x-x'}\right). \tag{10e}$$

5. The solution to the tunnelling problem

In this section we will investigate the solution to the action given in equations (9*a*)–(9*c*).

From the computation of the charge density velocity $U_c^{(-)}$ in equation (10*a*) we observe that in the limit $\hat{V}_{\|} \ll \hat{V}_{\perp}$ the antisymmetric velocity $U_c^{2(-)} < 0$ becomes negative. This means that there is no interchain charge density wave excitation. This condition occurs when $t_{\|}^2/J_{\|} \ll t_{\perp}^2/J_{\perp}$. Due to this we will consider explicitly two cases: (a) $\hat{V}_{\|} > \hat{V}_{\perp}$ and (b) $\hat{V}_{\|} \ll \hat{V}_{\perp}$.

5.1. The case $\hat{V}_{\|} > \hat{V}_{\perp}$

For this case the parameters given in equations (10*a*)–(10*c*) are well defined. Therefore we can use the action given in equations (9*a*)–(9*c*). The results obtained for this case are similar to the one obtained for two coupled Luttinger chains [9]. Investigating the Coulomb gas in equation (9*c*) one finds that the fugacity $\hat{t}_{\perp}$ is a relevant variable. Therefore one obtains a plasma phase with coherent tunnelling for charges between the chain. In the sense of the Luttinger liquid one finds that due to tunnelling the Luttinger liquid is destroyed. The interchain hopping variable obeys the scaling equation

$$\frac{\mathrm{d}t_{\perp}}{\mathrm{d}l} = (1-2\alpha)t_{\perp}. \tag{11}$$

α is the Fermi surface exponent which obeys $\alpha = \frac{1}{4}[2/g_c^{(-)} + g_c^{(-)}/2 - 2]$; since $g_c^{(-)} \leqslant 2$ it follows that $2\alpha < 1$ leading to $t_{\perp}(l) \to \infty$. Therefore the Luttinger liquid is destroyed and tunnelling takes place.

5.2. $V_{\|} \ll V_{\perp}$

For this case we find that the antisymmetric velocity satisfies $U_c^{2(-)} < 0$. (This case will occur when $t_{\perp}^2/J_{\perp} \gg t_{\|}^2/J_{\|}$.) In order to make use of the derivation presented in section 4 we will assume that the parameters in equation (10*a*) are such that the coefficient of the antisymmetric mode vanishes, $U_c^{(-)} = 0$. Therefore the Coulomb gas in equation (9*c*) must be replaced by a gas in one space dimension instead of 1+1 dimensions. Due to this dimensional reduction the Coulomb gas for the charge density Q_c will be in the confined phase. Therefore $Q_c = 0$. We will have only a two-dimensional Coulomb gas for the spin part!

Since $Q_c = 0$ it follows that $Q_s = 2Q_{\uparrow} = -2Q_{\downarrow}$. In the same way $S_s = 2S_{\uparrow} = -2S_{\downarrow}$. For the present case the Coulomb gas is replaced by

$$S^{(\mathrm{Coulomb})} \simeq \int \mathrm{d}x \int \mathrm{d}\tau \int \mathrm{d}x' \int \mathrm{d}\tau' \left\{ \left[\frac{1}{2g_s^{(-)}} q(x,\tau) q(x',\tau') + \frac{g_s^{(-)}}{8} s(x,\tau) s(x',\tau') \right] \hat{U}(x,t;x',t') \right\} \tag{12a}$$

where $g \equiv Q_s = \pm 2$, $s \equiv S_s = \pm 2$. The Coulomb gas in equation (12*a*) is controlled by the fugacity $\hat{J}_{\perp}$. The Coulomb gas in equation (12*a*) is analysed using the renormalization group method used in [6]–[8]. We find the scaling equation

$$\frac{\mathrm{d}J_{\perp}}{\mathrm{d}l} = \left[\left(\frac{2}{g_s^{(-)}} + \frac{g_s^{(-)}}{2} \right) - 2 \right] J_{\perp}. \tag{12b}$$

Since $g_s^{(-)} = 2$ it follows that $dJ_\perp/dl = 0$. This means that $J_\perp$ is marginal. As a result the present gas (case 5.2) is not in the plasma phase! Due to the fact that $dJ_\perp/dl = 0$ no gap will appear and the only effect of the Coulomb gas given in equation (12*b*) will be to renormalize the spinon parameters $U_s^{(-)} \to U_{s,R}^{(-)}$ and $g_s^{(-)} \to g_{s,R}^{(-)}$. We find a Luttinger liquid with a symmetric charge density wave $\Phi_c^{(+)}$ and two spin density waves, $\Phi_s^{(+)}$ symmetric and $\Phi_s^{(-)}$ antisymmetric mode. Those excitations are controlled by the effective action:

$$S \approx \int dx \int d\tau \left\{ \frac{U_c^{(+)}}{2g_c^{(+)}} \left[(\partial_x \Phi_c^{(+)})^2 + \frac{1}{2(U_c^{(+)})^2} (\partial_\tau \Phi_c^{(+)})^2 \right] + \frac{U_s^{(+)}}{2g_s^{(+)}} \left[(\partial_x \Phi_s^{(+)})^2 + \frac{1}{2(U_s^{(+)})^2} (\partial_\tau \Phi_s^{(+)})^2 \right] + \frac{U_s^{(-)}}{2g_s^{(-)}} \left[(\partial_x \Phi_s^{(-)})^2 + \frac{1}{2(U_s^{(-)})^2} (\partial_\tau \Phi_s^{(-)})^2 \right] \right\}. \tag{12c}$$

6. Conclusion

The two-coupled-spinon–holon model has been investigated. One finds that for the generic case the Luttinger liquid is destroyed.

For the particular case $t_\perp^2/J_\perp \gg t_\parallel^2/J_\parallel$ one finds that the Luttinger liquid is preserved and tunnelling is prohibited in agreement with Anderson's conjecture.

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